FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper G

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. A curve has the equation $y = (3x - 5)^3$.

(a) Find an equation for the tangent to the curve at the point P (2, 1). **(4)**

The tangent to the curve at the point Q is parallel to the tangent at P.

(b) Find the coordinates of Q. **(3)**

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2. *(a)* Use the identities for $\cos (A + B)$ and $\cos (A - B)$ to prove that

$$2 \cos A \cos B \equiv \cos (A + B) + \cos (A - B).$$ **(2)**

(b) Hence, or otherwise, find in terms of π the solutions of the equation

$$2 \cos (x + \tfrac{\pi}{2}) = \sec (x + \tfrac{\pi}{6}),$$

for x in the interval $0 \leq x \leq \pi$. **(7)**

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3. Differentiate each of the following with respect to x and simplify your answers.

(a) $\ln (\cos x)$ **(3)**

(b) $x^2 \sin 3x$ **(3)**

(c) $\dfrac{6}{\sqrt{2x-7}}$ **(4)**

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4. *(a)* Express $2 \sin x° - 3 \cos x°$ in the form $R \sin (x - \alpha)°$ where $R > 0$ and $0 < \alpha < 90$. **(4)**

(b) Show that the equation

$$\operatorname{cosec} x° + 3 \cot x° = 2$$

can be written in the form

$$2 \sin x° - 3 \cos x° = 1.$$ **(1)**

(c) Solve the equation

$$\operatorname{cosec} x° + 3 \cot x° = 2,$$

for x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place. **(5)**

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5. *(a)* Show that $(2x + 3)$ is a factor of $(2x^3 - x^2 + 4x + 15)$. **(2)**

(b) Hence, simplify

$$\frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15}.$$ **(4)**

(c) Find the coordinates of the stationary points of the curve with equation

$$y = \frac{2x^2 + x - 3}{2x^3 - x^2 + 4x + 15}.$$ **(6)**

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6. The population in thousands, P, of a town at time t years after 1st January 1980 is modelled by the formula

$$P = 30 + 50e^{0.002t}.$$

Use this model to estimate

(a) the population of the town on 1st January 2010, **(2)**

(b) the year in which the population first exceeds 84 000. **(4)**

The population in thousands, Q, of another town is modelled by the formula

$$Q = 26 + 50e^{0.003t}.$$

(c) Show that the value of t when $P = Q$ is a solution of the equation

$$t = 1000 \ln (1 + 0.08e^{-0.002t}).$$ **(3)**

(d) Use the iteration formula

$$t_{n+1} = 1000 \ln (1 + 0.08e^{-0.002t_n})$$

with $t_0 = 50$ to find t_1, t_2 and t_3 and hence, the year in which the populations of these two towns will be equal according to these models. **(4)**

Turn over

7.

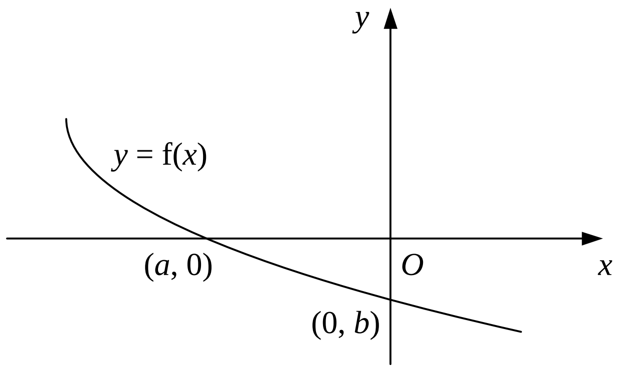

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$ which meets the coordinate axes at the points $(a, 0)$ and $(0, b)$, where a and b are constants.

(a) Showing, in terms of a and b, the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

(i) $y = \mathrm{f}^{-1}(x)$,

(ii) $y = 2\mathrm{f}(3x)$. **(6)**

Given that

$$\mathrm{f}(x) = 2 - \sqrt{x+9}, \quad x \in \mathbb{R}, \quad x \geq -9,$$

(b) find the values of a and b, **(3)**

(c) find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. **(5)**

END